

QUESTION 2013

Group - A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any ten of the following:

i) If $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$, then the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if

a) $\Delta > 0$

b) $\Delta < 0$

c) $\Delta = 0$

d) none of these

ii) If the matrix $\begin{pmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ \lambda & -3 & 0 \end{pmatrix}$ is singular then the value of λ is

- a) 0 b) 4 ✓ c) 2 d) -1

iii) If A be a matrix whose inverse exists then which of the following is not true?

- a) $(A^T)^{-1} = (A^{-1})^T$ ✓ b) $A^{-1} = (\det A)^{-1}$ c) $(A^2)^{-1} = (A^{-1})^2$ d) none of these

iv) $\frac{d}{dx}(e^{xy}) =$

- a) e^{xy} b) xe^{xy} ✓ c) ye^{xy} d) none of these

v) The degree of the function $f(x, y) = \tan^{-1} \frac{y}{x}$ is

- a) 1 b) 0 c) 2 ✓ d) none of these

vi) The inverse of the matrix $\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$ is

- ✓ a) $\frac{1}{3} \begin{pmatrix} 1 & -2 \\ 1 & -1 \end{pmatrix}$ b) $\begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix}$ c) $\frac{1}{3} \begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix}$ d) none of these

vii) The value of $\int \frac{dx}{x \log x}$ is

- a) $\log|x| + c$ ✓ b) $\log|\log x| + c$ c) $x \log|x| + c$ d) none of these

viii) If α, β and γ be the roots of the equation $x^3 + 7x - 2 = 0$ then $\sum \alpha^2 =$

- a) 0 b) 14 c) -14 ✓ d) 4

ix) Which of the following is a null set?

- a) $A = \{0\}$ b) $A = \{\phi\}$
 ✓ c) $A = \{x : x \text{ is an integer \& } 1 < x < 2\}$ d) none of these

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x) The value of $\lim_{x \rightarrow 0} \frac{\sin x}{x}$ (where x is radian) is

- ✓ a) 1 b) 0 c) ∞ d) -1

xi) The conic $\frac{l}{r} = 1 - e \cos \theta$ represents a parabola if

- ✓ a) $e = 1$ b) $e > 1$ c) $e < 1$ d) none of these

xii) What is the value of the following limit, $\lim_{x \rightarrow 0} (1+x)^{1/x}$?

- a) 1 ✓ b) e c) 0 d) none of these

Group - B

(Short Answer Type Questions)

2. Evaluate the integral $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx$.

See Topic: DEFINITE INTEGRALS, Short Answer Type Question No. 3.

3. Express $\begin{bmatrix} -3 & 4 & 1 \\ 2 & 3 & 0 \\ 1 & 4 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix. ✓

See Topic: MATRICES, Short Answer Type Question No. 6.

4. If $u = \tan^{-1} \frac{x+y}{\sqrt{x} + \sqrt{y}}$, then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{4} \sin 2u$.

See Topic: FUNCTIONS OF SEVERAL VARIABLES, Long Answer Type Question No. 3.

5. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ two of whose roots are in the ratio 3 : 2.

See Topic: POLYNOMIAL, Short Answer Type Question No. 16.

6. Prove that the set of real numbers of the form $a + b\sqrt{2}$ where a and b are rational numbers, forms a field under addition and multiplication.

See Topic: BINARY COMPOSITION, Short Answer Type Question No. 5.

Group - C

(Long Answer Type Questions)

7. a) State Descartes' rule of sign. Using this rule find the nature of the root of the equation $x^3 - 7x^2 + 21x - 21 = 0$.

b) Solve the following system of linear equations by Cramer's rule:

$$x - y + 2z = 1, \quad x + y + z = 2, \quad 2x - y + z = 5.$$

c) If by a transformation of rectangular axis to another with same origin the expression $ax + by$ changes to $a'x' + b'y'$, prove that $a^2 + b^2 = a'^2 + b'^2$.

a) See Topic: POLYNOMIAL, Long Answer Type Question No. 10.

b) See Topic: MATRICES, Long Answer Type Question No. 12.

c) See Topic: TRANSFORMATION OF CO-ORDINATES, Long Answer Type Question No. 11.

8. a) If G be a group such that $(ab)^2 = a^2b^2 \forall a, b \in G$, show that the group G is Abelian.

b) Show that $\int_0^1 \frac{\log(1+x)}{1+x^2} dx = \frac{\pi}{8} \log 2$.

c) If $y = e^{-x} \sin x$, then show that $y_4 + 4y = 0$.

a) See Topic: BINARY COMPOSITION, Short Answer Type Question No. 6.

b) See Topic: DEFINITE INTEGRALS, Short Answer Type Question No. 10.

c) See Topic: SUCCESSIVE DIFFERENTIATION, Short Answer Type Question No. 9.

9. a) Show that the matrix $A = \frac{1}{3} \begin{pmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ is orthogonal and hence find A^{-1} .

b) If $A = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$, then show that $A^2 - 2A + I_2 = O_2$. Hence obtain A^{-1} and also find A^{100} .

c) Reduce the following equation to the canonical form and determine the nature of the conic represented by it: $8x^2 - 12xy + 17y^2 + 16x - 12y + 3 = 0$.

a) & b) See Topic: MATRICES, Long Answer Type Question No. 13.

c) See Topic: GENERAL EQUATION OF SECOND DEGREE, Long Answer Type Question No. 1.

10. a) Solve the equation $x^3 - 3x^2 + 12x + 16 = 0$ by Cardan's method.

b) Prove that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.

c) If α, β, γ are the three roots of $x^3 + px^2 + qx + r = 0$, obtain the value of $\sum (\alpha - \beta)^2$.

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- a) See Topic: **POLYNOMIAL**, Long Answer Type Question No. 11.
- b) See Topic: **SET THEORY**, Short Answer Type Question No. 7.
- c) See Topic: **POLYNOMIAL**, Long Answer Type Question No. 6.

11. a) State Rolle's theorem. Examine whether Rolle's theorem is applicable or not for the function $f(x) = 1 - |x - 1|$, $\forall x \in [0, 2]$.

b) If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$, then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$.

c) Find for what values of x , the following expression is maximum and minimum respectively:

$$2x^3 - 21x^2 + 36x - 20.$$

- a) See Topic: **EXPANSION OF FUNCTIONS**, Short Answer Type Question No. 8.
- b) See Topic: **FUNCTIONS OF SEVERAL VARIABLES**, Long Answer Type Question No. 4.
- c) See Topic: **MISCELLANEOUS**, Long Answer Type Question No. 2.